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H. Pauwels[a]; G. de Mey[a]; C. Reynaerts[a]; F. Cuypers[a]
[a] Laboratory of Electronics, University of Ghent, Gent, Belgium

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One dimensional stationary states with constant electrical induction in ferroelectric liquid crystals

by H. PAUWELS, G. DE MEY, C. REYNAERTS and F. CUYPERS
Laboratory of Electronics, University of Ghent, Belgium,
Sint-Pietersnieuwstraat 41, B-9000 Gent, Belgium

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In a ferroelectric liquid crystal, it is the electrical induction, D, and not the electric field, E, which is independent of the space coordinate in a one dimensional analysis. This causes, at small applied voltages, stationary states which have a homogeneous orientation and are field free over most of the liquid crystal. At higher voltages homogeneous field states with zero, one or two surface layers may exist.

1. Introduction

Many researchers (e.g. [1–3]) have presented models for the calculation of stationary states and the switching behaviour of ferroelectric liquid crystals. They assume that the applied electric field is independent of the coordinate perpendicular to the surface. A typical order of magnitude of the electric field E is 10^6 V/m and of the permanent polarization density P is 30×10^{-5} C/m^2, which means that P is about 30 times larger than $\varepsilon_0 E$. Since, at least in one dimensional situations, it is the component of $\bar{D} = \varepsilon\varepsilon_0\bar{E} + \bar{P}$ normal to the surface which is independent of the space coordinate, the electric field may vary very strongly with this coordinate. In this paper we want to point out some of the consequences for stationary states. Domains play an important part in ferroelectric liquid crystals but they must be treated in a higher dimensionality. We shall limit ourselves here to one dimensional situations because then analytical solutions can be found.

2. The model

In a ferroelectric liquid crystal the molecules are ordered in layers perpendicular to the glass surfaces. The director $\bar{n}$ tilts away from the layer normal by a constant angle ϑ, but its azimuthal orientation is determined by an angle φ which also determines the orientation of the permanent polarization in the layer plane (see figure 1). The z axis is chosen perpendicular to the layers, the x axis perpendicular to the glass surfaces and the y axis parallel to these surfaces. In one dimensional situations the angle φ is an unknown function of x. In the so-called one constant approximation for the distortion energy, stationary states are given by solutions of the equation [1–7]

$$\alpha \frac{d^2\varphi}{dx^2} = EP \sin\varphi, \tag{1a}$$

with the boundary conditions

$$\alpha \frac{d\varphi}{dx} = +\gamma_1 \sin 2\varphi - \gamma_2 \sin\varphi, \quad x = 0 \tag{2a}$$

$$= -\gamma_1 \sin 2\varphi - \gamma_2 \sin\varphi, \quad x = d. \tag{2b}$$

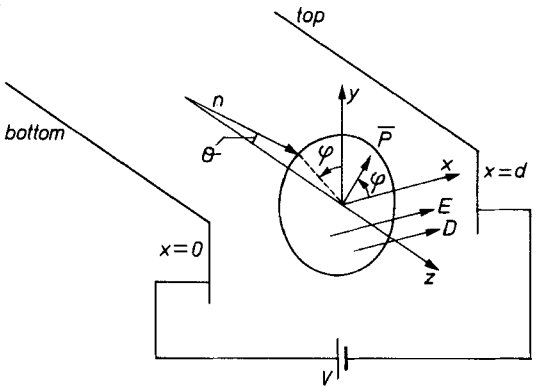

Figure 1. Configuration of a ferroelectric liquid crystal.

Some authors (e.g. [1]) claim that equation (1 *a*) is an approximation which is only justified if the energy of the self field of the polarization charge is dominated by the elastic contribution, which is the case when the sample thickness d is much smaller than the polarized self correlation length, $\sqrt{(\alpha/P^2)}$. We claim, together with two Japanese groups [4–7], that equations (1 *a*) is correct but that E should be calculated correctly. The validity of this equation is probably first explained in [6], where it is solved, mainly numerically, with the boundary condition $\varphi = 0$ at $x = 0$ and $\varphi = \pi$ at $x = d$. In [4, 5, 7] the dynamic equations are derived and solved numerically. In this paper we limit ourselves to stationary solutions but under general boundary conditions.

Most authors [1–3] take $E = +V/d$. This is however not correct since the electrical induction D is a constant and not the electric field E. Instead

$$E = \frac{D - P\cos\varphi}{\varepsilon\varepsilon_0}, \tag{1 b}$$

where D depends on the applied voltage by

$$\begin{aligned} V &= \int E dx \\ &= \frac{D}{\varepsilon\varepsilon_0} d - \frac{P}{\varepsilon\varepsilon_0} \int \cos\varphi \, dx. \end{aligned} \tag{1 c}$$

The integration of equations (1) is obtained by multiplying equation (1 *a*) with $d\varphi/dx$, leading to

$$\left.\begin{aligned} \frac{\varepsilon\varepsilon_0\alpha}{2} \frac{d}{dx}\left(\frac{d\varphi}{dx}\right)^2 &= (D - P\cos\varphi) \frac{d}{dx}(D - P\cos\varphi), \\ \varepsilon\varepsilon_0\alpha\left(\frac{d\varphi}{dx}\right)^2 &= a + (D - P\cos\varphi)^2, \end{aligned}\right\} \tag{3}$$

where a is an integration constant.

We shall start from the stable state $\varphi = 0$ obtained e.g. by applying a positive voltage. Then we apply a negative voltage of increasing magnitude, and look for stationary states in between the $\varphi = 0$ state and the stable state $\varphi = \pi$.

3. Field free homogeneous states φ_h.

From equation (1 *a*) it follows that homogeneous solutions where $\varphi = c^{st}$ are only possible if either $\sin\varphi = 0$, i.e. $\varphi = 0$ and $\varphi = \pi$, or if $\varepsilon\varepsilon_0 E = D - P\cos\varphi = 0$. We first look for the second type of solution. The behaviour of φ, $\cos\varphi$ and E are shown in figure 2. These solutions correspond with $a = 0$ in equation (3), such that

$$\alpha \frac{\partial\varphi}{\partial x} = \pm P \sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} (\cos\varphi - \cos\varphi_h), \tag{4}$$

where φ_h is the value of φ in the homogeneous part of the liquid crystal. The positive sign corresponds with the bottom surface layer, the negative sign with the top surface layer. The voltage drop over such a surface layer is

$$\begin{aligned} \int E dx &= \frac{P}{\varepsilon\varepsilon_0} \int (\cos\varphi_h - \cos\varphi) \frac{1}{d\varphi/dx} d\varphi, \\ &= \pm \sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} \int d\varphi, \\ &= - \sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} (\varphi_h - \varphi_{b\,or\,t}). \end{aligned} \tag{5 a}$$

The total voltage drop is therefore

$$V = - \sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} (2\varphi_h - \varphi_b - \varphi_t). \tag{5 b}$$

With increasing V, φ_h increases but also φ_b and φ_t until at threshold the weakest surface, in this case the bottom surface, can no longer hold φ. Following equation (2 *a*), this occurs at the maximum value of $\partial\varphi/\partial x$, i.e. at

$$\cos\varphi_b = \frac{\gamma}{8} + \sqrt{\left[\frac{1}{2} + \left(\frac{\gamma}{8}\right)^2\right]}, \quad \gamma = \gamma_2/\gamma_1. \tag{6 a}$$

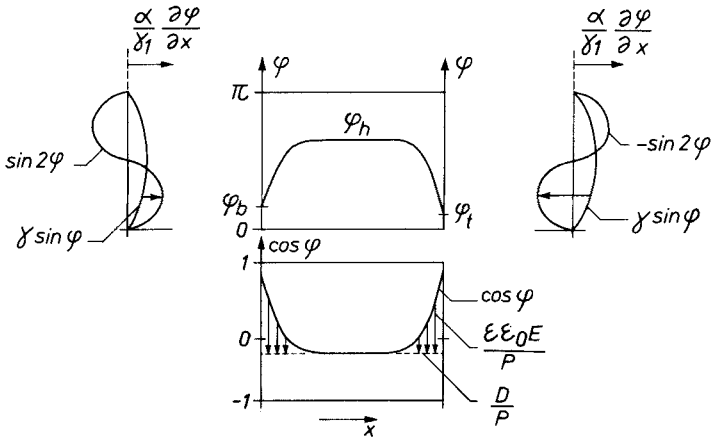

Figure 2. Orientation and field distribution in the field free homogeneous state φ_h. The anchoring conditions on the top and bottom electrodes are also shown.

The corresponding value of φ_h follows from equation (4)

$$\cos\varphi_h = \cos\varphi_b - \sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}\frac{\gamma_1}{P}(\sin 2\varphi_b - \gamma\sin\varphi_b) \tag{6b}$$

and the corresponding value of φ_t follows from equations (2*b*) and (4) as the solution of

$$(\sin 2\varphi_t + \gamma\sin\varphi_t)\sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}\frac{\gamma_1}{P} = \cos\varphi_t - \cos\varphi_h. \tag{6c}$$

The threshold voltage follows from equation (5*b*)

$$-\sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}V_t = 2\varphi_h - \varphi_b - \varphi_t. \tag{6d}$$

In figure 3 we have plotted this threshold voltage $(\varepsilon\varepsilon_0/\alpha)^{1/2}V_t$ versus $(\varepsilon\varepsilon_0/\alpha)^{1/2}\gamma_1/P$ for various values of γ. Typical values are $\varepsilon = 5, \alpha = 4 \times 10^{-12}$ N, $\gamma_1 = 3 \times 10^{-5}$ N/m, $P = 30 \times 10^{-5}$ C/m^2 and thus $(\varepsilon\varepsilon_0/\alpha)^{1/2} = 3{\cdot}3$ and $\gamma_1/P = 0{\cdot}1$. The threshold voltage is then normally well below 1 V, but for stronger surface anchoring or weaker polarization, it can reach a maximum of about 5 V.

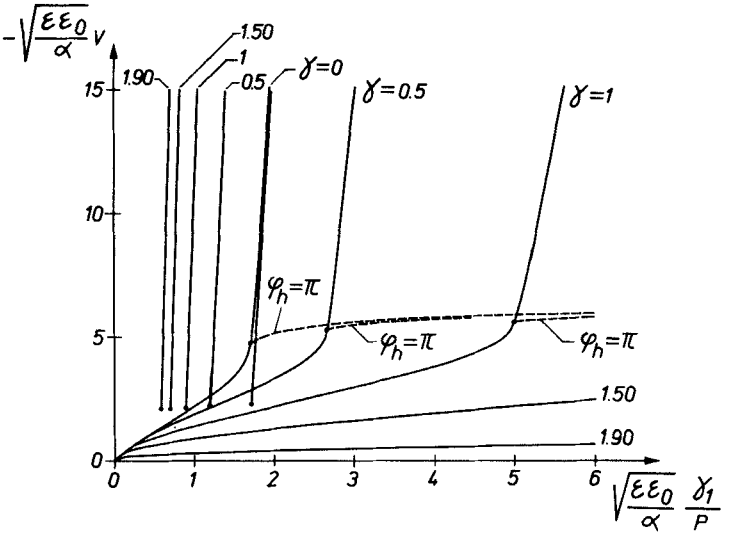

Figure 3. Threshold voltages for the field free homogeneous state (φ_h) and for the homogeneous field states with one (π_1) and two (π_2) surface layers.

For sufficiently strong surface anchoring, the threshold will not be reached before φ_h reaches π. The corresponding value of V is obtained from

$$-\sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}V = 2\pi - \varphi_b - \varphi_t, \tag{7a}$$

with

$$1 + \cos\varphi_b = \sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}\frac{\gamma_1}{P}(\sin 2\varphi_b - \gamma\sin\varphi_b), \tag{7b}$$

$$1 + \cos\varphi_t = \sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}\frac{\gamma_1}{P}(\sin 2\varphi_t + \gamma\sin\varphi_t). \tag{7c}$$

This curve is also shown on figure 3. The point where $\varphi_h = \pi$ is reached is indicated. Below both curves described by equations (6) and (7), the φ_h states occur. In figure 4 only a single value of γ, i.e. $\gamma = 0{\cdot}5$ is considered, so that the regions where the various states can occur are more clearly indicated. Notice that there is no Freedericksz transition: when φ_h, φ_b and φ_t go to zero, V also goes to zero. This is different from the constant E treatment of most other authors, e.g. [1], who obtain, for strong surface anchoring a Freedericksz threshold voltage $V_t = \pi^2\alpha/Pd$ which for $d = 2\,\mu$m has a typical value of 0·06 V.

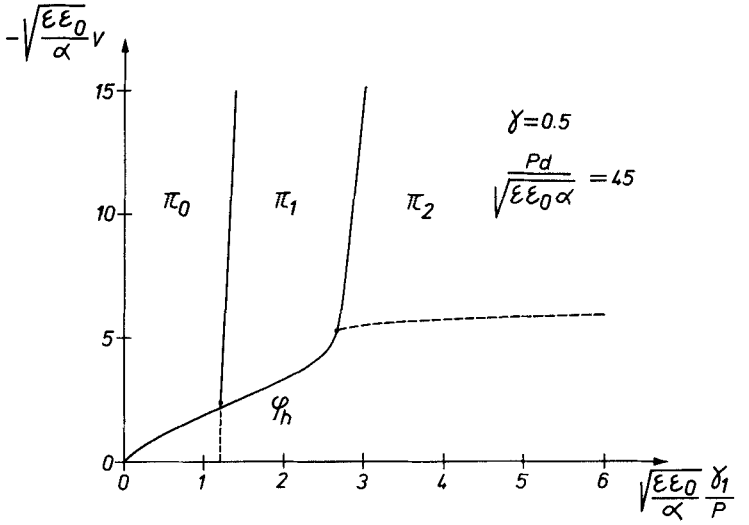

Figure 4. Regions for the φ_h, π_0, π_1 and π_2 states for $\gamma = 0{\cdot}5$. The π_0 state is the completely switched state with no surface layers.

If more detailed information about the surface layers is required, it can be noted that equation (4) can be integrated analytically. For the bottom layer we obtain

$$\frac{\sin\frac{1}{2}(\varphi_h - \varphi)}{\sin\frac{1}{2}(\varphi_h - \varphi_b)} \cdot \frac{\sin\frac{1}{2}(\varphi_h + \varphi_b)}{\sin\frac{1}{2}(\varphi_h + \varphi)} = \exp\left[-\frac{xP}{\sqrt{(\varepsilon\varepsilon_0\alpha)}}\sin\varphi_h\right]. \tag{8a}$$

For $\varphi_h = \pi$, this solution reduces to

$$\operatorname{tg}\frac{\varphi}{2} - \operatorname{tg}\frac{\varphi_b}{2} = \frac{xP}{\sqrt{(\varepsilon\varepsilon_0\alpha)}}. \tag{8b}$$

4. Homogeneous field states π_1 and π_2

If below threshold φ_h reaches π with increasing V, we obtain a homogeneous state in which the field differs from zero, with two surface layers. We shall call it the π_2 state. Above threshold φ_h may switch to π, but the stronger surface layer may still hold. Again the field is different from zero in the homogeneous region. This state will be called the π_1 state. Both cases corresponding to $a \neq 0$ in equation (3). For simplicity of notation let us analyse a surface layer near the bottom electrode (see figure 5).

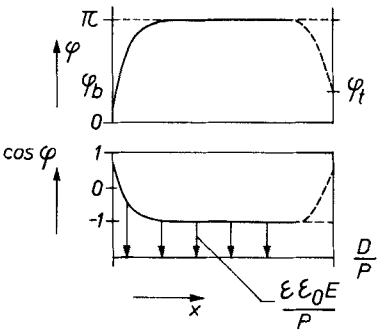

Figure 5. Orientation and field distribution in the homogeneous field states.

Equation (3) now reads

$$\varepsilon\varepsilon_0\alpha\left(\frac{d\varphi}{dx}\right)^2 = (D - P\cos\varphi)^2 - (D + P)^2,$$

$$= +4P^2\cos^2\frac{\varphi}{2}\left(-\frac{D}{P} - \sin^2\frac{\varphi}{2}\right). \tag{9}$$

The voltage over the single surface layer is

$$V = \int E dx = \frac{D + P}{\varepsilon\varepsilon_0} d + \int_{\varphi_b}^{\pi} \frac{(D - P\cos\varphi) - (D + P)}{\varepsilon\varepsilon_0} \frac{d\varphi}{(d\varphi/dx)},$$

$$= \frac{D + P}{\varepsilon\varepsilon_0} d - \sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} \int_{\varphi_b}^{\pi} \frac{2d\sin\varphi/2}{\sqrt{[-(D/P) - \sin^2\varphi/2]}},$$

$$= \frac{D + P}{\varepsilon\varepsilon_0} d - 2\sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} \operatorname{Arcsin}\frac{1}{\sqrt{(-D/P)}} + 2\sqrt{\left(\frac{\alpha}{\varepsilon\varepsilon_0}\right)} \operatorname{Arcsin}\frac{\sin\varphi_b/2}{\sqrt{(-D/P)}}. \tag{10}$$

If more detailed information is needed, equation (9) can again be integrated analytically:

$$\frac{P}{\sqrt{(\varepsilon\varepsilon_0\alpha)}} x = \int_{\varphi_b}^{\varphi} \frac{d\sin\varphi/2}{[1 - \sin^2\varphi/2)]\sqrt{[-(D/P) - \sin^2\varphi/2]}},$$

$$= \frac{1}{\sqrt{[-(D/P) - 1]}} \ln \frac{\sqrt{[-(D/P) - \sin^2\varphi/2]} + \sqrt{[-(D/P) - 1]}\sin\varphi/2}{\sqrt{(-D/P)}\cos\varphi/2}\Bigg|_{\varphi_b}^{\varphi}. \tag{11}$$

If $D + P \to 0$, we again obtain equation (8 *b*).

The threshold for the double surface layer is obtained as follows. Equation (6 *a*) gives φ_b, from equation (2 *a*) and equation (9) we calculate $-D/P$

$$-\frac{D}{P} = \sin^2\frac{\varphi_b}{2}\left[1 + \frac{\varepsilon\varepsilon_0}{\alpha}\frac{\gamma_1^2}{P^2}(2\cos\varphi_b - \gamma)^2\right];$$

from equation (2*b*) and equation (9) we obtain φ_t through

$$-\frac{D}{P} = \sin^2\frac{\varphi_t}{2}\left[1 + \frac{\varepsilon\varepsilon_0}{\alpha}\frac{\gamma_1^2}{P^2}(2\cos\varphi_t + \gamma)^2\right].$$

Finally, from equation (10) but for two surface layers, we obtain for the threshold voltage

$$-\sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}V_t = \frac{Pd}{\sqrt{(\varepsilon\varepsilon_0\alpha)}}\left(1 + \frac{D}{P}\right) - 4\,\mathrm{Arcsin}\frac{1}{\sqrt{(-D/P)}} + 2\,\mathrm{Arcsin}\frac{\sin\varphi_b/2}{\sqrt{(-D/P)}}$$

$$+ 2\,\mathrm{Arcsin}\frac{\sin\varphi_t/2}{\sqrt{(-D/P)}}. \tag{12}$$

This result is also shown in figures 3 and 4; the transition $\varphi_h = \pi$ and $D + P = 0$ is indicated. For $Pd/\sqrt{(\varepsilon\varepsilon_0\alpha)}$ we have taken a value of 45.

The threshold for the single surface layer is obtained when the strongest surface can no longer hold, i.e. for

$$\cos\varphi_t = -\frac{\gamma}{8} + \sqrt{\left[\frac{1}{2} + \left(\frac{\gamma}{8}\right)^2\right]}.$$

From equation (2*b*) and equation (9) we calculate $-D/P$

$$-\frac{D}{P} = \sin^2\frac{\varphi_t}{2}\left[1 + \frac{\varepsilon\varepsilon_0}{\alpha}\frac{\gamma_1^2}{P^2}(2\cos\varphi_t + \gamma)^2\right];$$

and finally from equation (10) we obtain for the threshold voltage

$$-\sqrt{\left(\frac{\varepsilon\varepsilon_0}{\alpha}\right)}V_t = \frac{Pd}{\sqrt{(\varepsilon\varepsilon_0\alpha)}}\left(1 + \frac{D}{P}\right) - 2\,\mathrm{Arcsin}\frac{1}{\sqrt{(-D/P)}} + 2\,\mathrm{Arcsin}\frac{\sin(\varphi_t/2)}{\sqrt{-D/P}}. \tag{13}$$

This threshold is shown also on figures 3 and 4.

In figure 4 for the case $\gamma = 0{\cdot}5$ the various regions are indicated where, respectively, the homogeneous field free state φ_h exists, the completely switched state π_0, the single layer homogeneous field state π_1 and the double layer homogeneous field state π_2. Because of the steepness of the threshold curves described by equations (12) and (13), we can say approximately that there exists a threshold value for γ_1, below which we switch directly from the φ_h state to the π_0 state, and above which we switch from the φ_h state to the π_1 state. The same holds true for the π_2 state.

5. Conclusions

Many calculations of one dimensional stationary states in ferroelectric liquid crystals are based on the assumption that the electric field is constant throughout the material. This is, however, not correct since it is the electrical induction which is constant. This causes some differences. First of all, there is no Freedericksz transition. As soon as the voltage is different from zero, a homogeneous state occurs which is field free except for the two surface layers. At sufficiently large voltage this state switches to a homogeneous state with either no, one or two surface layers, depending on the strength of the surface anchoring.

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